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Low Level Representations for E_{10} and E_{11}.

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ABSTRACT. We work out the decomposition of the indefinite Kac Moody algebras E_{10} and E_{11} w.r.t. their respective subalgebras A_9 and A_{10} at low levels. Tables of the irreducible representations with their outer multiplicities are presented for E_{10} up to level $\ell = 18$ and for E_{11} up to level $\ell = 10$. On the way we confirm and extend existing results for E_{10} root multiplicities, and for the first time compute non-trivial root multiplicities of E_{11}.

1. Introduction

In this contribution, we will be mainly concerned with the indefinite Kac Moody algebras E_{10} and E_{11} (see [**19, 31**] for the basic theory of Kac Moody algebras). Of these E_{10} is hyperbolic (all its proper regular subalgebras are either finite or affine), while E_{11} is not (its regular subalgebra E_{10} is neither finite nor affine). Consideration of these two algebras is chiefly motivated by recent attempts to find out more about the still unknown symmetries underlying M-Theory. Following the discovery of a Kac-Moody billiard and arithmetical chaos in superstring cosmology in [**9**], a novel one-dimensional σ-model based on the formal coset $E_{10}/K(E_{10})$ was proposed in [**10**], with $K(E_{10})$ the maximal compact subgroup of E_{10}, and a precise identification was made in a BKL-type expansion between the bosonic supergravity fields at a given space point and their first spatial gradients on the one hand, and the first three levels of E_{10} on the other (the emergence of E_{10} in the dimensional reduction of $D=11$ supergravity[**8**] to one dimension had been conjectured already long ago [**18**]). An alternative proposal was made in [**35, 36**], where the rank-11 algebra E_{11} is conjectured to be the fundamental symmetry of M-Theory. It is likewise based on a non-linear realization of the relevant "Kac-Moody group", but differs from [**10**] in that it puts more emphasis on spacetime covariance. The results of the present paper may serve to discriminate between the two proposals, as our tables show very explicitly where E_{11} deviates from E_{10} (namely by all those representations in Table 2, for which the last label m^{10} is different from zero).

As in [**10**] (as well as in [**33, 7, 28, 35, 36**]) our investigation is based on a decomposition of these algebras w.r.t. to their maximal $sl(n,\mathbb{R})$ subalgebras, namely the two algebras $A_9 \equiv sl(10,\mathbb{R})$ and $A_{10} \equiv sl(11,\mathbb{R})$, both of which can be enlarged to the Lie algebras of the full general linear groups $GL(10,\mathbb{R})$ and $GL(11,\mathbb{R})$ within E_{10} and E_{11}, respectively, by inclusion of the "exceptional" Cartan subalgebra generator. This decomposition is organized by means of the level ℓ, which is the number of times a certain distinguished "exceptional" root α_0 appears in the decomposition of a given root α [**10**]. The level-0 sectors then consist of the A_9 and A_{10} subalgebras themselves. For the first three levels, the representations which appear are of the same type for all E_n with $n \geq 9$. The Dynkin labels of the

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relevant representations are

$$
\begin{aligned}
\ell = 1 \quad &: \quad (001000000\dots) \\
\ell = 2 \quad &: \quad (000001000\dots) \\
\ell = 3 \quad &: \quad (100000010\dots)
\end{aligned}
\tag{1.1}
$$

corresponding to the antisymmetric tensors with three and six indices for $\ell = 1, 2$, respectively, and a mixed Young tableau for $\ell = 3$; therefore, the associated Lie algebra generators are $E^{a_1 a_2 a_3}, E^{a_1 \cdots a_6}$ and $E^{a_0|a_1 \cdots a_8}$ (with the constraint $E^{[a_0|a_1 \cdots a_8]} = 0$). In the physical interpretation, the first two of these are associated with the electric and magnetic excitations of the 3-form field of maximal $D = 11$ supergravity, while the third can be viewed as dual to the graviton [**33, 7, 16, 35, 10**]. For the finite dimensional Lie algebras E_n with $n \leq 8$, there are no higher level representations beyond $\ell = 3$; in particular, for $n = 8$, the level-3 representation collapses to the (1000000) of A_7, and for E_7 and E_6 it is absent altogether. For $n \geq 9$, there are representations at all levels. While these are easy to classify for E_9, a veritable explosion in the number of representations takes place for $n \geq 10$, and this will be our main focus here. For the hyperbolic algebra E_{10}, we will obtain the complete list of A_9 representations up to level 18; for E_{11}, we give them up to level 10. In principle, these lists can be extended considerably further[1].

There is by now a substantial body of work on root multiplicities of low rank hyperbolic KM algebras, and the corresponding decompositions of these algebras w.r.t. to certain distinguished affine subalgebras, starting with the rank 3 hyperbolic algebra studied [**11**] where closed formulas were derived for multiplicities of roots of affine level ≤ 2. This work was extended up to affine level 5 and to other low rank algebras in [**4**]-[**6**] and [**21**]-[**25**], [**26, 27**]. A closed formula for the E_{10} root multiplicities at affine level two was derived in [**20**] (affine levels zero and one being trivial). The decomposition w.r.t. an affine subalgebra has the advantage of classifying an infinite number of Lie algebra elements, but only "close to the lightcone", and appears rather hopeless beyond the very first levels. By contrast, the decomposition w.r.t. to a finite dimensional algebra yields only a finite amount of information at each step, but can be carried to rather high levels. Besides, it is of special interest for possible applications in string and M-Theory because it yields a concrete realization of the algebra in terms of irreducible representations of subgroups that have a direct physical interpretation. To penetrate the structure of E_{10} even further one can try to "intertwine" the representation theory of A_9 and E_9, or to exploit the decomposition under its principal $sl(2, \mathbb{R})$ subalgebra [**15, 32**]. As for E_{11}, and apart from the $\ell \leq 3$ results of [**35, 36**] (which do not require knowledge of non-trivial E_{11} root multiplicities), our results are completely new; in particular, this is the first time that non-trivial E_{11} root multiplicities have been calculated.

While our methods are thus by no means sufficient to arrive at a completely explicit and manageable realization of these indefinite KM algebras, the results of

[1] Actually, the tables are now available up to level $\ell = 28$ for E_{10}, where we have already 3276 inequivalent irreducible representations of $SL(10, \mathbb{R})$ with maximum outer multiplicity equal to 46450629 for the representation (110100011). Evident lack of space prevents us from presenting the complete results here. However, all the raw data are included in the `arXiv.org` source package of this work and can be found at `http://www.arxiv.org/e-print/hep-th/0301017`. This package also contains the decomposition of the Feingold-Frenkel algebra AE_3 in terms of A_2 representations up to level $\ell = 56$. Code to calculate such decompositions has been made publicly available [**12**].

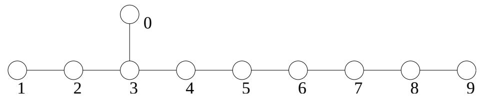

FIGURE 1. E_{10}

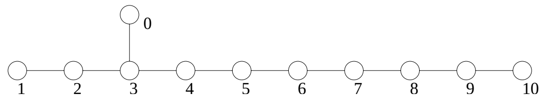

FIGURE 2. E_{11}

this paper can nevertheless serve as useful "raw data" for future investigations. At the very least, they give a very concrete flavor of the stunning complexity of these Lie algebras: readers should keep in mind that the tables presented here represent only the very first steps into a mathematical structure, whose complexity will forever continue to grow with increasing level as one moves deeper and deeper into the lightcone. Even restricting attention to the low level representations exhibited here, the task of working out, say, the Lie algebra structure constants beyond the very first levels seems too daunting even to contemplate. From the physics perspective, the possible link noticed in [**10**] between the three infinite towers of representations (2.19) and the spatial gradients of the bosonic fields of $D = 11$ supergravity is a tantalizing hint, but the task of finding a physical interpretation for all the other representations remains a major challenge. Realizing the full E_{10} (or E_{11}) symmetry probably requires *a theory beyond* $D = 11$ supergravity, as it seems very unlikely that these other representations simply correspond to auxiliary or Stückelberg-type degrees of freedom. For the time being, however, we must refer readers to the still unknown chapter in THE BOOK [**1**] for an answer to these and other questions!

2. Roots and Dynkin labels

The indefinite Kac Moody algebras E_{10} and E_{11} are characterized by the Dynkin diagrams displayed in figs.1 and 2. The simple roots of the respective A_n subalgebras will be labeled $\alpha_1, \alpha_2, \dots$ (counting from left to right); the "exceptional" root connected to α_3 will be designated by α_0. Because both algebras are simply laced, the associated Cartan matrices are $A_{ij} = (\alpha_i|\alpha_j)$.

As in [**10**] we will decompose both algebras into irreducible representations of their finite dimensional subalgebras A_9 and A_{10}, respectively. This corresponds to a slicing of the forward lightcone in the respective root lattices of E_{10} and E_{11} by spacelike hyperplanes; hence the sections will be ellipsoidal with *finitely many roots in each slice*. By contrast, slicing the lightcone by lightlike or timelike hyperplanes would result in a decomposition w.r.t. an affine or an indefinite subalgebra, in both of which cases each slice contains *infinitely many roots*. As already mentioned the corresponding $SL(n+1,\mathbb{R})$ groups are enlarged to $GL(n+1,\mathbb{R})$, respectively, by inclusion of the Cartan generator h_0 associated with the exceptional root.

For both algebras any positive root can be expressed as

$$\alpha = \ell\alpha_0 + \sum_{j=1}^{n} m^j \alpha_j \tag{2.2}$$

with non-negative integers ℓ and m^j, and $n = 9$ or $n = 10$ for E_{10} and E_{11}, respectively. The number ℓ is the "level" and counts the number of occurrences of the exceptional root α_0 in α. It must not be confused with the "affine level" $(= m^9)$ used in the decomposition of E_{10} w.r.t. its affine subalgebra E_9 [**11, 20**] (there appears to be no useful analog of the affine level for E_{11}).

The irreducible representations (or "modules" in more mathematical language) of A_9 and A_{10} are completely characterized their Dynkin labels, which are 9-tuples $(p_1 \dots p_9)$ and 10-tuples $(p_1 \dots p_{10})$ of non-negative integers, respectively, such that the corresponding highest weights λ are given by

$$\lambda = \sum_{j=1}^{n} p_j \lambda^j \tag{2.3}$$

for $n = 9, 10$. Here λ^j are the fundamental weights of A_n obeying $(\alpha_i|\lambda^j) = \delta_i^j$ with $i, j = 1, \dots, n$ for $n = 9, 10$, respectively. At the same time, p_j is equal to the number of columns with j vertical boxes in the associated Young tableau. The fundamental weights of A_n are explicitly given by $\lambda^i = \sum_{j=1}^{n} S^{ij}\alpha_j$, where S^{ij} is the inverse Cartan matrix of A_n (see below). We use small Greek letters for the weights of A_n to distinguish them from the weights of E_{n+1}, whose fundamental weights Λ^j (for $j = 0, 1, \dots, n$) must in addition satisfy $(\alpha_0|\Lambda^i) = \delta_0^i$ and differ by a vector orthogonal to the simple roots of A_n, see below. The explicit relation between the coefficients m^j and the Dynkin labels will be derived below.

Each root α comes with a certain root multiplicity $\mathrm{mult}(\alpha)$ counting the number of independent Lie algebra elements associated with α [**19**]. These Lie algebra elements will be split up according to which representation of A_9 or A_{10} they belong to. Any root α will therefore also appear as a weight in certain representations $\mathcal{R}$, with corresponding weight multiplicities $\mathrm{mult}_{\mathcal{R}}(\alpha)$. Furthermore, any such representation can appear several times, and the number of times a representation occurs within a given level will be called the *outer multiplicity*; the outer multiplicity of an admissible representation (i.e. a representation satisfying the two conditions (2.12) and (2.17) below) may be zero. We thus have the following picture: the finitely many roots at a given level ℓ are superpositions of certain A_n weight diagrams, each one of which may appear several times with a certain outer multiplicity.

While the relevant representations that can appear are not hard to find, the determination of the outer multiplicities is a difficult and as yet unsolved problem. Knowing them in closed form would be equivalent to a complete knowledge of the root multiplicities for indefinite and hyperbolic Kac Moody algebras (there is so far not a single example of such an algebra for which the root multiplicities are known in closed form) and would take us a considerable way towards the ultimate goal of finding an explicit and manageable realization of the full algebra. All we know is that typically the root multiplicities increase exponentially with their length $(-\alpha^2)$ [**19**], implying similar growth properties for the outer multiplicities.

In fact, the formulas restricting the possible representations that can appear at level ℓ are easy to deduce [**10**] and also easy to solve, especially with the help of a computer. To begin with, the representations which appear at level $\ell + 1$ must be contained in the set of representations obtained by taking the product of the level one representation (001000000) with all the representations occurring at level ℓ. This is because the level-$(\ell + 1)$ generators are generally obtained by commuting the level-ℓ generators with a level one generator, and this is also how one quickly derives the representations given in formula (1.1). However, many of the representations obtained in this way will drop out, and the method becomes more and more impractical with higher levels.

A better and much quicker way to derive the admissible representations is by judicious analysis of the corresponding highest weights. Indeed, the formula linking the coefficients m^j and the Dynkin labels of the relevant A_9 or A_{10} is quite straightforward to deduce. First of all, we notice that the Chevalley generator f_0 is a highest weight state w.r.t. A_9 or A_{10}, with Dynkin label $(001000000\dots)$ because

$$\begin{aligned} h_i(f_0) &\equiv [h_i\,, f_0] = \delta_{i3} f_0 \\ e_i(f_0) &\equiv [e_i\,, f_0] = 0 \end{aligned} \tag{2.4}$$

for $i = 1, \dots, n$. The associated highest weight is $\Lambda = -\alpha_0$. Other highest weight states associated with roots $\Lambda \equiv -\alpha = -\alpha_{j_1} - \cdots - \alpha_{j_k}$ are built from multiple commutators

$$f_{j_1\dots j_k} := [f_{j_1}, \dots [f_{j_{k-1}}, f_{j_k}] \dots] \tag{2.5}$$

by permuting the Chevalley generators f_j in all possible ways and taking suitable linear combinations

$$f^{(\Lambda)} = \sum c^{(\Lambda)}_{j_1\dots j_k} f_{j_1\dots j_k} \tag{2.6}$$

which are annihilated by the adjoint action of all e_i for $i = 1, \dots, n$, i.e.

$$e_i(f^{(\Lambda)}) \equiv [e_i, f^{(\Lambda)}] = 0 \tag{2.7}$$

The level of $f^{(\Lambda)}$ is the number of j_ν with $j_\nu = 0$. The A_n module is then built by acting on this state with the Chevalley generators f_i, where again $i = 1, \dots, n$. The weight of $f^{(\Lambda)}$ follows from

$$[h_i, f_{j_1\dots j_k}] = \left(-\sum_{\nu=1}^{k} A_{ij_\nu} \right) f_{j_1\dots j_k} = p_i f_{j_1\dots j_k} \tag{2.8}$$

An explicit basis of the module $V(\Lambda)$ is given in [**29**]: all states can be represented in the form

$$[f_{m_1}, ..., [f_{m_s}, f_{j_1\dots j_k}]...] = f_{m_1\dots m_s j_1\dots j_k} \quad , \quad m_\nu \in \{1, \dots, n\} \tag{2.9}$$

with weight labels

$$q_j = p_j - \sum_{\nu=1}^{s} A_{jm_\nu} \tag{2.10}$$

where the m_ν are ordered according to the prescription given in [**29**] (which is somewhat cumbersome to disentangle for larger rank).

Setting $\alpha = \alpha_{j_1} + \cdots + \alpha_{j_k} = \ell\alpha_0 + m^1\alpha_1 + \cdots + m^n\alpha_n$ and writing out the relation $p_i = -\sum_\nu A_{ij_\nu}$ we get (for $n = 9, 10$)

$$\begin{aligned}
p_1 &= m_2 - 2m_1 \\
p_2 &= m_1 + m_3 - 2m_2 \\
p_3 &= \ell + m_2 + m_4 - 2m_3 \\
p_4 &= m_3 + m_5 - 2m_4 \\
&\cdot \\
&\cdot \\
&\cdot \\
p_{n-1} &= m_{n-2} + m_n - 2m_{n-1} \\
p_n &= m_{n-1} - 2m_n
\end{aligned} \tag{2.11}$$

Because this is a highest weight state by assumption, the corresponding weight $\sum p_j\lambda^j$ is dominant, hence $p_j \geq 0$. Inverting this formula, we obtain [**10**]

$$m^i = S^{i3}\ell - \sum_{j=1}^{n} S^{ij}p_j \tag{2.12}$$

where S^{ij} are the inverse Cartan matrices of A_9 and A_{10}, respectively. This formula also allows us to express the highest weight $\Lambda = -\alpha$ in terms of the A_n weights: multiplying by α_i and summing over i, we get

$$\sum_{j=1}^{n} m^j\alpha_j = \ell\sum_{j=1}^{n} S^{3j}\alpha_j - \sum_{j=1}^{n} p_j\lambda^j \tag{2.13}$$

where we have also used the definition of the fundamental A_n weights. Hence,

$$\Lambda \equiv -\alpha = -\ell\Big(\alpha_0 + \sum_{j=1}^{n} S^{3j}\alpha_j\Big) + \sum_{j=1}^{n} p_j\lambda^j \tag{2.14}$$

Observe that $p_j = (\alpha_j|\Lambda) = (\alpha_j|\lambda)$ for $j = 1, \ldots, n$ because the first term has vanishing scalar product with these simple roots.

The inverse Cartan matrices are, respectively, given by

$$S^{ij}[A_9] = \frac{1}{10}\begin{pmatrix}
9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\
8 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 \\
7 & 14 & 21 & 18 & 15 & 12 & 9 & 6 & 3 \\
6 & 12 & 18 & 24 & 20 & 16 & 12 & 8 & 4 \\
5 & 10 & 15 & 20 & 25 & 20 & 15 & 10 & 5 \\
4 & 8 & 12 & 16 & 20 & 24 & 18 & 12 & 6 \\
3 & 6 & 9 & 12 & 15 & 18 & 21 & 14 & 7 \\
2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 8 \\
1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9
\end{pmatrix} \tag{2.15}$$

for A_9 and

$$S^{ij}[A_{10}] = \frac{1}{11}\begin{pmatrix} 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 \\ 9 & 18 & 16 & 14 & 12 & 10 & 8 & 6 & 4 & 2 \\ 8 & 16 & 24 & 21 & 18 & 15 & 12 & 9 & 6 & 3 \\ 7 & 14 & 21 & 28 & 24 & 20 & 16 & 12 & 8 & 4 \\ 6 & 12 & 18 & 24 & 30 & 25 & 20 & 15 & 10 & 5 \\ 5 & 10 & 15 & 20 & 25 & 30 & 24 & 18 & 12 & 6 \\ 4 & 8 & 12 & 16 & 20 & 24 & 28 & 21 & 14 & 7 \\ 3 & 6 & 9 & 12 & 15 & 18 & 21 & 24 & 16 & 8 \\ 2 & 4 & 6 & 8 & 10 & 12 & 14 & 16 & 18 & 9 \\ 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \end{pmatrix} \tag{2.16}$$

for A_{10}. Because the entries of S^{ij} are all positive, and because m^i and p_i must all be non-negative integers, the formula (2.12) strongly constrains the possible representations for each ℓ.

A further constraint derives from the fact that the highest weight $\Lambda = -\alpha$ must be a root; this means that its norm must obey $\Lambda^2 \leq 2$ [2].This requirement effectively implements the Serre relations. For E_{10} it implies

$$\Lambda^2 = \alpha^2 = \sum_{i,j=1}^{9} p_i S^{ij} p_j - \tfrac{1}{10}\ell^2 \implies \sum_{i,j=1}^{9} p_i S^{ij} p_j \leq 2 + \tfrac{1}{10}\ell^2 \tag{2.17}$$

For E_{11}, we have the bound

$$\Lambda^2 = \alpha^2 = \sum_{i,j=1}^{10} p_i S^{ij} p_j - \tfrac{2}{11}\ell^2 \implies \sum_{i,j=1}^{10} p_i S^{ij} p_j \leq 2 + \tfrac{2}{11}\ell^2 \tag{2.18}$$

As we will see this constraint again eliminates many further representations which would still be compatible with (2.12).

An important infinite series of E_{10} Lie algebra elements was identified in [**10**] by searching for "affine representations", namely those highest weights Λ for which $m^9 = 0$ in (2.2). There are three infinite towers of E_{10} Lie algebra elements at levels $\ell = 3k+1, 3k+2$ and $3k+3$ (for $k \geq 0$) with Dynkin labels

$$\begin{aligned} \ell = 3k+1 \quad &: \quad (00100000k) \\ \ell = 3k+2 \quad &: \quad (00000100k) \\ \ell = 3k+3 \quad &: \quad (10000001k) \end{aligned} \tag{2.19}$$

In our tables, these representations are always listed at the very top. Inserting the Dynkin labels into the above formula we see that the associated highest weights obey $\Lambda^2 = 2$ for all k; since $\mathrm{mult}(\alpha) = 1$ for $\alpha^2 = 2$, the outer multiplicities are always one. Explicitly, the Lie algebra elements are

$$E_{a_1 \ldots a_k}{}^{b_1 b_2 b_3} \quad , \quad E_{a_1 \ldots a_k}{}^{b_1 \ldots b_6} \quad , \quad E_{a_1 \ldots a_k}{}^{b_0 | b_1 \ldots b_8} \tag{2.20}$$

and were tentatively associated in [**10**] to the spatial gradients of the bosonic fields of $D = 11$ supergravity and their duals. There exist three similar infinite towers of conjugate (or "transposed") Lie algebra elements obtained by acting with the Chevalley involution on the above elements. These affine representations are distinguished because they contain the affine subalgebra E_9. To see this more explicitly,

[2]This is only a necessary but not a sufficient criterion for Λ to be a root, see our discussion of the E_{11} roots in the next section.

we note that the elements of the latter must commute with the central charge of E_9. This requirement is equivalent to imposing a "dimensional reduction" on the elements (2.20) by setting $a_1 = \cdots = a_k = 10$ and taking the remaining indices $b_j \in \{1, \ldots, 9\}$. Using the decomposition $\mathbf{248} \rightarrow \mathbf{80} \oplus \mathbf{84} \oplus \overline{\mathbf{84}}$ of the adjoint of E_8 under its $SL(9, \mathbb{R})$ subgroup, the relevant Lie algebra elements are then obtained from this truncated set and the generators of the subgroup $GL(9, \mathbb{R}) \subset GL(10, \mathbb{R})$; the central charge generator c is the remaining diagonal generator of $GL(10, \mathbb{R})$.

A second infinite series of admissible affine A_9 representations at levels $\ell = 3k$ is characterized by the Dynkin labels

$$\ell = 3k \quad : \quad (00000000k) \tag{2.21}$$

It is easy to see that these representations must have outer multiplicity zero. The associated roots are

$$\alpha \equiv k\Big(3\alpha_0 + \sum_{j=1}^{9} m^j \alpha_j\Big) = k\delta \tag{2.22}$$

where the root δ associated with the values $(m^1 \ldots m^9) = (246543210)$ is the affine null vector of E_9. However, there can be no such generator in E_9, because all the E_9 elements are already contained in the three infinite towers (2.19) and their transposed towers as we have just shown.

At low levels the "disappearance" of certain representations can be understood as a consequence of the Jacobi identity, which implies that any commutator of the form

$$\Big[x_1, \ldots, \big[x_k, [E^{[a_1 a_2 a_3}, [E^{a_4 a_5 a_6}, E^{a_7 a_8 a_9]}]]\big] \ldots\Big] \tag{2.23}$$

must vanish for arbitrary Lie algebra elements x_j. For instance, it is easy to see that the representation (000000001) drops out by virtue of (2.23). At higher levels, unfortunately, the situation is rather less transparent, and it does not appear that (2.23) is of much use in eliminating representations, except when it corresponds to a highest weight state for A_n.

For E_{11}, there are three similar infinite towers with labels

$$\begin{aligned} \ell = 3k+1 \quad &: \quad (00100000k0) \\ \ell = 3k+2 \quad &: \quad (00000100k0) \\ \ell = 3k+3 \quad &: \quad (10000001k0) \end{aligned} \tag{2.24}$$

and again $\Lambda^2 = 2$ with outer multiplicity one for all these representations. Now, the Lie algebra elements are given by

$$E_{[a_1 b_1] \ldots [a_k b_k]}{}^{c_1 c_2 c_3} \quad , \quad E_{[a_1 b_1] \ldots [a_k b_k]}{}^{c_1 \ldots c_6} \quad , \quad E_{[a_1 b_1] \ldots [a_k b_k]}{}^{c_0 | c_1 \ldots c_8} \tag{2.25}$$

with antisymmetric index pairs $[a_1 b_1]$, etc. Obviously the representations (2.19) can again be obtained from those of (2.24) by dimensional reduction (i.e. setting $c_i = 11$ and taking $a_i, b_i \in \{1, \ldots, 10\}$). However, a physical interpretation of these E_{11} elements analogous to the one suggested in [**10**] for the representations (2.19) appears no longer possible.

3. Determination of low level A_n representations

We next explain how to work out the higher level representations that appear in the decompositions of E_{10} and E_{11} w.r.t. their respective A_n subalgebras. Our results for the representations up to level $\ell = 18$ for E_{10} and up to level $\ell = 10$ for

E_{11} are displayed in the tables following this section. The first step in determining the representations is to exploit the two conditions (2.12) and (2.17), which are already quite restrictive by themselves, because the n-tuples $(m^1 ... m^n)$ and $(p_1 \dots p_n)$ must both consist of non-negative integers. Hence (2.12) and (2.17) admit only a finite number of solutions for given ℓ, both for E_{10} and E_{11}. Furthermore, with the help of a computer these solutions are easy to find up to very high levels.

For given ℓ, each highest weight Λ generates a representation with a corresponding finite weight diagram $P(\Lambda)$ of A_n, which belongs to an elliptic slice of the forward lightcone in the root lattice of E_{n+1}. The weights $\lambda \in P(\Lambda)$ come with certain weight multiplicities which we designate by $\mathrm{mult}_{\mathcal{R}}(\lambda)$ for the representation $\mathcal{R} \equiv \mathcal{R}(\Lambda)$. Hence the roots α at a given level make up a "stack" of weight diagrams, such that the root multiplicity $\mathrm{mult}(\alpha)$ of α as a root of E_{n+1} is the sum of the multiplicities of α as a weight occurring in the various weight diagrams $P(\Lambda)$. Thus the following formula linking the E_{n+1} root multiplicities with the A_n weight multiplicities is self-evident

$$\mathrm{mult}(\alpha) = \sum_{i=1}^{n_\ell} \mu_\ell\Big(\mathcal{R}_i^{(\ell)}\Big)\ \mathrm{mult}_{\mathcal{R}_i^{(\ell)}}(\alpha) \tag{3.26}$$

where $\mathcal{R}_i^{(\ell)}$ is the i-th representation at level ℓ, and the index $i = 1, \dots, n_\ell$ counts the level-ℓ representations. The numbers $\mu_\ell(\mathcal{R}_i^{(\ell)})$ are the outer multiplicities at level ℓ: they count how many times the representation $\mathcal{R}_i^{(\ell)}$ occurs in the decomposition at level ℓ. As far as we know, there does not exist an analog of the Peterson recursion formula for the outer multiplicities, which is why we have to take the detour via (3.26) for their determination.

The only quantities entering formula (3.26) which are known in full generality, although somewhat cumbersome to work out for more complicated representations, are the weight multiplicities $\mathrm{mult}_{\mathcal{R}}(\lambda)$ for the A_n representations $\mathcal{R}$. Given an A_n module $\mathcal{R}(\Lambda)$ with highest weight Λ and any weight $\lambda \in P(\Lambda)$ (which implies $\lambda < \Lambda$) the weight multiplicity $\mathrm{mult}_{\mathcal{R}(\Lambda)}(\lambda)$ follows from the Freudenthal recursion formula (see for instance [**17**])

$$\begin{aligned} &\Big((\Lambda|\Lambda) + 2\,\mathrm{ht}\,\Lambda - (\lambda|\lambda) - 2\,\mathrm{ht}\,\lambda\Big)\,\mathrm{mult}_{\mathcal{R}(\Lambda)}(\lambda) \\ &\qquad = 2\sum_{\alpha>0}\sum_{k\geq 1}(\lambda + k\alpha|\alpha)\ \mathrm{mult}_{\mathcal{R}(\Lambda)}(\lambda + k\alpha) \end{aligned} \tag{3.27}$$

where the first sum on the r.h.s. ranges over all positive roots of A_n. For the calculation we employ a COMMON LISP implementation of an improved version of this formula, using the algorithm described in [**30, 34**]. To determine the outer multiplicities of the relevant A_n modules from the E_{n+1} root multiplicities (as far as they are known), we proceed by the method of exhaustion, starting with the highest weights obeying $\Lambda^2 = 2$ whose associated representations always come with outer multiplicity one if Λ is a real root (this is always true for E_{10}, but there may be "spurious real roots" for E_{11}, see below). Next, we descend the weight system until we hit a weight $\alpha \equiv \lambda \in P(\Lambda)$ whose multiplicity as a root of E_{n+1} exceeds its multiplicity as a weight in $P(\Lambda)$. This root, then, is a highest weight in a new representation. Consequently, any highest weight Λ_1 for which $\Lambda_1^2 < \Lambda_2^2$ may appear as a weight in the highest weight module of Λ_2. However, highest weights Λ and Λ' of the same length associated with different labels $(p_1 \dots p_n)$ and $(p'_1 \dots p'_n)$

can never appear as weights in each other's weight diagram, as this would require $p_i \leq p'_i$ with strict inequality for at least one i, in contradiction with the assumed equality of the norms, implying $\sum p_i S^{ij} p_j = \sum p'_i S^{ij} p'_j$.

The E_{n+1} root multiplicities can be calculated in principle from the Peterson recursion formula (see [**19**], Exercise 11.12). Unfortunately, this formula becomes more and more impractical with increasing height due to the large number of root decompositions one has to take into account, see below. For E_{10}, closed formulas exist up to affine level 2 [**20**] (following earlier results for the rank 3 case in [**11**]). An implicit formula for the E_{10} root multiplicities at affine level three was derived in [**3**], but we have not been able to extract any numbers from it. All E_{10} root multiplicities up to affine level 6 and height 231 have been tabulated in [**2**]. For E_{11}, on the other hand, tables of root multiplicities have been lacking in the literature so far.

Because the E_{10} root multiplicities listed in [**2**] are a crucial ingredient in our calculation, we have performed an independent check based on a "brute force" evaluation of the Peterson recursion formula rather than the Weyl-orbit method employed there (whose description in the appendix of [**2**] seems somewhat obscure in retrospect), confirming and extending the tables of [**2**][3]. The Peterson formula reads

$$\Big((\alpha|\alpha) - 2\,\mathrm{ht}\,(\alpha)\Big)c_\alpha = \sum_{\substack{\beta',\beta''>0\\ \beta'+\beta''=\alpha}} (\beta'|\beta'')c_{\beta'}c_{\beta''} \tag{3.28}$$

where

$$c_\beta := \sum_{k\geq 1} \frac{1}{k}\mathrm{mult}\left(\frac{\beta}{k}\right) \tag{3.29}$$

and thus requires the decomposition of a given root Λ as a sum of two terms, both of which are linear combinations of the simple roots with non-negative coefficients (but not necessarily roots themselves). For E_{10} this procedure boils down to separating the possible β's into two classes. The first consists of the positive real roots β or their integer multiples up to a given height, which is half the height of the root Λ whose multiplicity we wish to compute. A complete list of these can be generated from the simple roots by applying height increasing Weyl reflections (every positive real root can be reached by such reflections, cf. Proposition 5.1.e of [**19**]), and then taking all their integer multiples. The second set consists of the positive imaginary roots up to the given height. For E_{10}, this calculation is greatly simplified by noting that for imaginary roots the minimal height element in any Weyl orbit belongs to the fundamental Weyl chamber. So we start from the positive linear combinations of the fundamental weights (see [**20**]) below the given height, and again apply height increasing Weyl reflections until we reach the given height. It is then easy (for the computer, at least) to enumerate all the possible combinations and to evaluate their contribution to the r.h.s. of (3.28). We note that the determination of the outer multiplicities via the method described above provides a consistency check on the root multiplicities obtained in this way. Namely, we have checked "experimentally" that in general one gets negative outer multiplicities if the wrong root multiplicities are used in (3.26).

[3] Which are now available up to height 320.

For E_{11} matters are more involved, first of all because so far no tables of E_{11} root multiplicities have been available, and secondly because of some additional subtleties which have no counterpart in E_{10} [4]. The latter are mainly due to the fact that the fundamental weight dual to the doubly overextended root α_{10} is spacelike and is no longer a root because it has positive *and* negative fractional coefficients when expressed in terms of the simple roots, as well as a disconnected support on the Dynkin diagram; explicitly,

$$\Lambda_{10} = \tfrac{3}{2}\alpha_0 + \alpha_1 + 2\alpha_2 + 3\alpha_3 + \tfrac{5}{2}\alpha_4 + 2\alpha_5 + \tfrac{3}{2}\alpha_6 + \alpha_7 + \tfrac{1}{2}\alpha_8 - \tfrac{1}{2}\alpha_{10} \tag{3.30}$$

The remaining fundamental weights are timelike, except for Λ_9 which is lightlike (and in fact the same as for E_{10}), and also have fractional coefficients which are, however, all positive with connected support on the Dynkin diagram. A new phenomenon is the appearance of spurious "real roots": for instance, by acting on $2\Lambda_{10}$ with suitable Weyl reflections we can easily produce combinations which look like real roots (having positive integer coefficients and connected support on the Dynkin diagram), but in fact are not! In listing the solutions to (2.12) and (2.17) we must watch out for such spurious "real roots": the corresponding solutions in the table appear with vanishing E_{11} root multiplicity, and therefore also vanishing outer multiplicity (again, negative outer multiplicities would result if this subtlety were not taken into account). To be on the safe side we have applied brute force once more in evaluating (3.28) by making lists of decompositions up to the full height of the root (and not just half the height as for E_{10}), whose multiplicity we are interested in. The relevant E_{11} root multiplicities are listed in the penultimate column of Table 2. Of course, the E_{11} root multiplicities for the "hyperbolic roots" (obeying $m^{10} = 0$) coincide with their multiplicities as E_{10} roots. In contradistinction to E_{10}, the root multiplicities start depending on the direction of α already for $\alpha^2 = -2$, where we have two Weyl orbits, with multiplicities 44 and 46, respectively (for higher rank algebras with more than one lightlike fundamental weight, this may already happen for null roots). Another noteworthy feature is that the E_{11} root multiplicities tend to be less than the number of transversal polarizations, in contrast to E_{10} where they are equal or greater. For instance, the null roots of E_{11}, all of which belong to a single Weyl orbit (and its integer multiples) have multiplicity equal to 8 instead of the number of transversal polarizations of a photon in eleven dimensions (=9), that one would expect on the basis of a vertex operator construction of such algebras [**13**].

For the outer multiplicities, we also note some general features. First of all, the outer multiplicities of the A_n representations in E_{n+1} at level $\ell+1$ usually will be strictly less than the outer multiplicities appearing in the product of the representations at level ℓ with the level one representation (001000000). Secondly, there are very few representations that have vanishing outer multiplicities; e.g. for E_{10}, there are none at levels $\ell = 7, 11, 13, 14$ and 17. Thirdly, up to the maximum level we have checked, all the A_9 representations in E_{10} with vanishing outer multiplicity have highest weights Λ obeying $\Lambda^2 = 0$. Apart from the spurious "real roots" this is also true for E_{11}. In other words, if a representation can appear, it will appear, usually with an outer multiplicity that increases with the level ℓ.

In the first set of tables we list the A_9 representations appearing in the decomposition of E_{10} up to and including level $\ell = 18$ in lexicographic order $\{m^j\}$

[4]Properties of these doubly overextended indefinite Kac-Moody algebras are discussed in [**14**], where they are called "very extended".

(starting from the back). The level is indicated in the first column, and in the second and third columns we give the Dynkin labels and the coefficients m^j appearing in (2.2). The norms Λ^2 of the associated highest weights are given in the fourth column. In the fifth column we list the dimension of the A_9 representation $\mathcal{R}(\Lambda) \equiv \mathcal{R}_{(p_1 \dots p_9)}$, which are computed by means of the Weyl dimension formula

$$\dim \mathcal{R}(\Lambda) = \prod_{\alpha > 0} \frac{(\Lambda + \rho|\alpha)}{(\rho|\alpha)} \tag{3.31}$$

where ρ is the A_9 Weyl vector such that the product $(\rho|\alpha)$ is just the height $\sum_j m^j$ of α as a root of A_9. The next column gives the multiplicity $\mathrm{mult}(\alpha)$ of the highest weight $\Lambda \equiv -\alpha$ as a root of E_{10}, taken from [**20**] and [**2**] (and checked again by our independent calculation). The last column contains our main result, namely the outer multiplicities of the associated A_9 representations in E_{10}. The E_{11} tables are organized similarly.

4. Acknowledgments

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Table 1: A_9 representations in E_{10} up to level $\ell = 18$

ℓ	p	m									Λ^2	dim $\mathcal{R}(\Lambda)$	mult(Λ)	μ
1	(001000000)	0	0	0	0	0	0	0	0	0	2	120	1	1
2	(000001000)	1	2	3	2	1	0	0	0	0	2	210	1	1
3	(100000010)	1	3	5	4	3	2	1	0	0	2	440	1	1
	(000000001)	2	4	6	5	4	3	2	1	0	0	10	8	0
4	(001000001)	2	4	6	5	4	3	2	1	0	2	1155	1	1
	(200000000)	1	4	7	6	5	4	3	2	1	2	55	1	1
	(010000000)	2	4	7	6	5	4	3	2	1	0	45	8	0
5	(000001001)	3	6	9	7	5	3	2	1	0	2	1848	1	1
	(100100000)	2	5	8	6	5	4	3	2	1	2	1848	1	1
	(000010000)	3	6	9	7	5	4	3	2	1	0	252	8	0
6	(100000011)	3	7	11	9	7	5	3	1	0	2	3200	1	1
	(000000002)	4	8	12	10	8	6	4	2	0	0	55	8	0
	(010001000)	3	6	10	8	6	4	3	2	1	2	8250	1	1
	(100000100)	3	7	11	9	7	5	3	2	1	0	1155	8	1
	(000000010)	4	8	12	10	8	6	4	2	1	-2	45	44	1
7	(001000002)	4	8	12	10	8	6	4	2	0	2	6160	1	1
	(000100100)	4	8	12	9	7	5	3	2	1	2	19800	1	1
	(110000010)	3	7	12	10	8	6	4	2	1	2	13860	1	1
	(001000010)	4	8	12	10	8	6	4	2	1	0	4950	8	1
	(200000001)	3	8	13	11	9	7	5	3	1	0	540	8	1
	(010000001)	4	8	13	11	9	7	5	3	1	-2	440	44	2
	(100000000)	4	9	14	12	10	8	6	4	2	-4	10	192	1
8	(000001002)	5	10	15	12	9	6	4	2	0	2	9240	1	1
	(000000200)	5	10	15	12	9	6	3	2	1	2	4950	1	1
	(100010010)	4	9	14	11	8	6	4	2	1	2	83160	1	1
	(000001010)	5	10	15	12	9	6	4	2	1	0	6930	8	1
	(011000001)	4	8	13	11	9	7	5	3	1	2	31185	1	1
	(100100001)	4	9	14	11	9	7	5	3	1	0	17280	8	2
	(000010001)	5	10	15	12	9	7	5	3	1	-2	2310	44	2
	(210000000)	3	8	14	12	10	8	6	4	2	2	1485	1	1
	(020000000)	4	8	14	12	10	8	6	4	2	0	825	8	0
	(101000000)	4	9	14	12	10	8	6	4	2	-2	990	44	2
	(000100000)	5	10	15	12	10	8	6	4	2	-4	210	192	2
9	(100000012)	5	11	17	14	11	8	5	2	0	2	14300	1	1
	(000000003)	6	12	18	15	12	9	6	3	0	0	220	8	0
	(010000110)	5	10	16	13	10	7	4	2	1	2	130680	1	1
	(100000020)	5	11	17	14	11	8	5	2	1	0	7920	8	1
	(001010001)	5	10	15	12	9	7	5	3	1	2	184800	1	1
	(200001001)	4	10	16	13	10	7	5	3	1	2	90090	1	1
	(010001001)	5	10	16	13	10	7	5	3	1	0	71280	8	2
	(100000101)	5	11	17	14	11	8	5	3	1	-2	9450	44	4
	(000000011)	6	12	18	15	12	9	6	3	1	-4	330	192	3
	(110100000)	4	9	15	12	10	8	6	4	2	2	46200	1	1
	(001100000)	5	10	15	12	10	8	6	4	2	0	13860	8	1
	(200010000)	4	10	16	13	10	8	6	4	2	0	11880	8	1
	(010010000)	5	10	16	13	10	8	6	4	2	-2	9240	44	3
	(100001000)	5	11	17	14	11	8	6	4	2	-4	1980	192	4
	(000000100)	6	12	18	15	12	9	6	4	2	-6	120	727	4
10	(001000003)	6	12	18	15	12	9	6	3	0	2	24024	1	1

	(000100020)	6	12	18	14	11	8	5	2	1	2	136125	1	1
	(000011001)	6	12	18	14	10	7	5	3	1	2	228096	1	1
	(101000101)	5	11	17	14	11	8	5	3	1	2	786500	1	1
	(000100101)	6	12	18	14	11	8	5	3	1	0	155925	8	2
	(110000011)	5	11	18	15	12	9	6	3	1	0	99099	8	3
	(001000011)	6	12	18	15	12	9	6	3	1	-2	35200	44	4
	(200000002)	5	12	19	16	13	10	7	4	1	-2	2925	44	3
	(010000002)	6	12	19	16	13	10	7	4	1	-4	2376	192	4
	(100110000)	5	11	17	13	10	8	6	4	2	2	228096	1	1
	(000020000)	6	12	18	14	10	8	6	4	2	0	19404	8	0
	(020001000)	5	10	17	14	11	8	6	4	2	2	136125	1	1
	(101001000)	5	11	17	14	11	8	6	4	2	0	155925	8	2
	(000101000)	6	12	18	14	11	8	6	4	2	-2	29700	44	3
	(300000100)	4	11	18	15	12	9	6	4	2	2	24024	1	1
	(110000100)	5	11	18	15	12	9	6	4	2	-2	35200	44	4
	(001000100)	6	12	18	15	12	9	6	4	2	-4	12375	192	6
	(200000010)	5	12	19	16	13	10	7	4	2	-4	2376	192	4
	(010000010)	6	12	19	16	13	10	7	4	2	-6	1925	727	10
	(100000001)	6	13	20	17	14	11	8	5	2	-8	99	2472	8
	(000000000)	7	14	21	18	15	12	9	6	3	-10	1	7749	3
11	(000001003)	7	14	21	17	13	9	6	3	0	2	34320	1	1
	(000000120)	7	14	21	17	13	9	5	2	1	2	43560	1	1
	(100001101)	6	13	20	16	12	8	5	3	1	2	943800	1	1
	(000000201)	7	14	21	17	13	9	5	3	1	0	35640	8	1
	(010100011)	6	12	19	15	12	9	6	3	1	2	1801800	1	1
	(100010011)	6	13	20	16	12	9	6	3	1	0	566280	8	3
	(000001011)	7	14	21	17	13	9	6	3	1	-2	46200	44	4
	(201000002)	5	12	19	16	13	10	7	4	1	2	237160	1	1
	(011000002)	6	12	19	16	13	10	7	4	1	0	163800	8	2
	(100100002)	6	13	20	16	13	10	7	4	1	-2	90090	44	5
	(000010002)	7	14	21	17	13	10	7	4	1	-4	11880	192	5
	(010011000)	6	12	19	15	11	8	6	4	2	2	914760	1	1
	(100002000)	6	13	20	16	12	8	6	4	2	0	124740	8	1
	(002000100)	6	12	18	15	12	9	6	4	2	2	453750	1	1
	(200100100)	5	12	19	15	12	9	6	4	2	2	823680	1	1
	(010100100)	6	12	19	15	12	9	6	4	2	0	609840	8	2
	(100010100)	6	13	20	16	12	9	6	4	2	-2	184800	44	5
	(000001100)	7	14	21	17	13	9	6	4	2	-4	13860	192	5
	(120000010)	5	11	19	16	13	10	7	4	2	2	200200	1	1
	(201000010)	5	12	19	16	13	10	7	4	2	0	189540	8	2
	(011000010)	6	12	19	16	13	10	7	4	2	-2	130680	44	5
	(100100010)	6	13	20	16	13	10	7	4	2	-4	71280	192	10
	(000010010)	7	14	21	17	13	10	7	4	2	-6	9240	727	11
	(210000001)	5	12	20	17	14	11	8	5	2	-2	14300	44	4
	(020000001)	6	12	20	17	14	11	8	5	2	-4	7920	192	5
	(101000001)	6	13	20	17	14	11	8	5	2	-6	9450	727	15
	(000100001)	7	14	21	17	14	11	8	5	2	-8	1980	2472	14
	(300000000)	5	13	21	18	15	12	9	6	3	-4	220	192	2
	(110000000)	6	13	21	18	15	12	9	6	3	-8	330	2472	9
	(001000000)	7	14	21	18	15	12	9	6	3	-10	120	7749	12
12	(100000013)	7	15	23	19	15	11	7	3	0	2	49280	1	1
	(000000004)	8	16	24	20	16	12	8	4	0	0	715	8	0
	(001001011)	7	14	21	17	13	9	6	3	1	2	3963960	1	1

	(200000111)	6	14	22	18	14	10	6	3	1	2	1034880	1	1
	(010000111)	7	14	22	18	14	10	6	3	1	0	823680	8	3
	(100000021)	7	15	23	19	15	11	7	3	1	-2	47190	44	4
	(000200002)	7	14	21	16	13	10	7	4	1	2	630630	1	1
	(110010002)	6	13	21	17	13	10	7	4	1	2	2851200	1	1
	(001010002)	7	14	21	17	13	10	7	4	1	0	926640	8	2
	(200001002)	6	14	22	18	14	10	7	4	1	0	444675	8	2
	(010001002)	7	14	22	18	14	10	7	4	1	-2	351000	44	6
	(100000102)	7	15	23	19	15	11	7	4	1	-4	45045	192	9
	(000000012)	8	16	24	20	16	12	8	4	1	-6	1485	726	7
	(000110100)	7	14	21	16	12	9	6	4	2	2	1905750	1	1
	(110001100)	6	13	21	17	13	9	6	4	2	2	3468465	1	1
	(001001100)	7	14	21	17	13	9	6	4	2	0	1161600	8	2
	(200000200)	6	14	22	18	14	10	6	4	2	0	240240	8	1
	(010000200)	7	14	22	18	14	10	6	4	2	-2	190575	44	4
	(101100010)	6	13	20	16	13	10	7	4	2	2	3753750	1	1
	(000200010)	7	14	21	16	13	10	7	4	2	0	490050	8	1
	(110010010)	6	13	21	17	13	10	7	4	2	0	2196480	8	3
	(001010010)	7	14	21	17	13	10	7	4	2	-2	711480	44	6
	(200001010)	6	14	22	18	14	10	7	4	2	-2	331695	44	6
	(010001010)	7	14	22	18	14	10	7	4	2	-4	261360	192	11
	(100000110)	7	15	23	19	15	11	7	4	2	-6	31185	727	17
	(000000020)	8	16	24	20	16	12	8	4	2	-8	825	2472	8
	(021000001)	6	12	20	17	14	11	8	5	2	2	405405	1	1
	(102000001)	6	13	20	17	14	11	8	5	2	0	330330	8	2
	(300100001)	5	13	21	17	14	11	8	5	2	2	316800	1	1
	(110100001)	6	13	21	17	14	11	8	5	2	-2	424710	44	7
	(001100001)	7	14	21	17	14	11	8	5	2	-4	126720	192	9
	(200010001)	6	14	22	18	14	11	8	5	2	-4	107250	192	10
	(010010001)	7	14	22	18	14	11	8	5	2	-6	83160	727	21
	(100001001)	7	15	23	19	15	11	8	5	2	-8	17280	2472	30
	(000000101)	8	16	24	20	16	12	8	5	2	-10	990	7747	22
	(220000000)	5	12	21	18	15	12	9	6	3	2	19305	1	1
	(030000000)	6	12	21	18	15	12	9	6	3	0	9075	8	0
	(301000000)	5	13	21	18	15	12	9	6	3	0	17160	8	1
	(111000000)	6	13	21	18	15	12	9	6	3	-4	21120	192	7
	(002000000)	7	14	21	18	15	12	9	6	3	-6	4950	727	7
	(200100000)	6	14	22	18	15	12	9	6	3	-6	9240	727	11
	(010100000)	7	14	22	18	15	12	9	6	3	-8	6930	2472	17
	(100010000)	7	15	23	19	15	12	9	6	3	-10	2310	7749	27
	(000001000)	8	16	24	20	16	12	9	6	3	-12	210	22725	18
13	(001000004)	8	16	24	20	16	12	8	4	0	2	76440	1	1
	(000010111)	8	16	24	19	14	10	6	3	1	2	2972970	1	1
	(101000021)	7	15	23	19	15	11	7	3	1	2	3929310	1	1
	(000100021)	8	16	24	19	15	11	7	3	1	0	786500	8	2
	(100101002)	7	15	23	18	14	10	7	4	1	2	9960720	1	1
	(000011002)	8	16	24	19	14	10	7	4	1	0	1081080	8	2
	(020000102)	7	14	23	19	15	11	7	4	1	2	3153150	1	1
	(101000102)	7	15	23	19	15	11	7	4	1	0	3659040	8	3
	(000100102)	8	16	24	19	15	11	7	4	1	-2	720720	44	6
	(300000012)	6	15	24	20	16	12	8	4	1	2	297000	1	1
	(110000012)	7	15	24	20	16	12	8	4	1	-2	436590	44	7
	(001000012)	8	16	24	20	16	12	8	4	1	-4	154440	192	10

	(200000003)	7	16	25	21	17	13	9	5	1	-4	11550	192	5
	(010000003)	8	16	25	21	17	13	9	5	1	-6	9360	726	10
	(000002100)	8	16	24	19	14	9	6	4	2	2	609840	1	1
	(100100200)	7	15	23	18	14	10	6	4	2	2	5662800	1	1
	(000010200)	8	16	24	19	14	10	6	4	2	0	653400	8	1
	(100020010)	7	15	23	18	13	10	7	4	2	2	5148000	1	1
	(011001010)	7	14	22	18	14	10	7	4	2	2	14594580	1	1
	(100101010)	7	15	23	18	14	10	7	4	2	0	7297290	8	3
	(000011010)	8	16	24	19	14	10	7	4	2	-2	784080	44	5
	(210000110)	6	14	23	19	15	11	7	4	2	2	3963960	1	1
	(020000110)	7	14	23	19	15	11	7	4	2	0	2162160	8	2
	(101000110)	7	15	23	19	15	11	7	4	2	-2	2502500	44	8
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	(300000020)	6	15	24	20	16	12	8	4	2	0	163800	8	1
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	(001000020)	8	16	24	20	16	12	8	4	2	-6	84700	727	16
	(010200001)	7	14	22	17	14	11	8	5	2	2	3243240	1	1
	(201010001)	6	14	22	18	14	11	8	5	2	2	6403320	1	1
	(011010001)	7	14	22	18	14	11	8	5	2	0	4290000	8	3
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	(101001001)	7	15	23	19	15	11	8	5	2	-4	1321320	192	18
	(000101001)	8	16	24	19	15	11	8	5	2	-6	249480	727	22
	(300000101)	6	15	24	20	16	12	8	5	2	-2	194040	44	5
	(110000101)	7	15	24	20	16	12	8	5	2	-6	283140	727	32
	(001000101)	8	16	24	20	16	12	8	5	2	-8	99000	2472	40
	(200000011)	7	16	25	21	17	13	9	5	2	-8	17160	2472	29
	(010000011)	8	16	25	21	17	13	9	5	2	-10	13860	7747	53
	(100000002)	8	17	26	22	18	14	10	6	2	-12	540	22712	32
	(003000000)	7	14	21	18	15	12	9	6	3	2	108900	1	1
	(120100000)	6	13	22	18	15	12	9	6	3	2	566280	1	1
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	(100200000)	7	15	23	18	15	12	9	6	3	-4	110880	192	6
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	(101010000)	7	15	23	19	15	12	9	6	3	-6	160380	727	21
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	(110001000)	7	15	24	20	16	12	9	6	3	-8	57750	2472	34
	(001001000)	8	16	24	20	16	12	9	6	3	-10	19800	7749	42
	(200000100)	7	16	25	21	17	13	9	6	3	-10	6160	7749	36
	(010000100)	8	16	25	21	17	13	9	6	3	-12	4950	22725	58
	(100000010)	8	17	26	22	18	14	10	6	3	-14	440	63085	57
	(000000001)	9	18	27	23	19	15	11	7	3	-16	10	167116	22
14	(000001004)	9	18	27	22	17	12	8	4	0	2	105105	1	1
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	(010010102)	8	16	25	20	15	11	7	4	1	2	24279255	1	1
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(000010003)	9	18	27	22	17	13	9	5	1	-6	45045	726	12
(100000300)	8	17	26	21	16	11	6	4	2	2	990990	1	1
(010002010)	8	16	25	20	15	10	7	4	2	2	12509640	1	1
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(400000002)	6	16	26	22	18	14	10	6	2	0	37125	8	1
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(101000002)	8	17	26	22	18	14	10	6	2	-10	50050	7747	68
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	(000002000)	9	18	27	22	17	12	9	6	3	-10	13860	7749	25
	(310000100)	6	15	25	21	17	13	9	6	3	0	524160	8	2
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	(011000100)	8	16	25	21	17	13	9	6	3	-8	317625	2472	45
	(100100100)	8	17	26	21	17	13	9	6	3	-10	168960	7749	83
	(000010100)	9	18	27	22	17	13	9	6	3	-12	20790	22725	66
	(400000010)	6	16	26	22	18	14	10	6	3	-2	30030	44	3
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	(020000010)	8	16	26	22	18	14	10	6	3	-10	33880	7747	49
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	(010000000)	9	18	28	24	20	16	12	8	4	-18	45	425227	59
15	(100000014)	9	19	29	24	19	14	9	4	0	2	143000	1	1
	(000000005)	10	20	30	25	20	15	10	5	0	0	2002	8	0
	(010000031)	9	18	28	23	18	13	8	3	1	2	1849848	1	1
	(001000202)	9	18	27	22	17	12	7	4	1	2	13513500	1	1
	(000110012)	9	18	27	21	16	12	8	4	1	2	24972948	1	1
	(110001012)	8	17	27	22	17	12	8	4	1	2	48262500	1	1
	(001001012)	9	18	27	22	17	12	8	4	1	0	16216200	8	3
	(200000112)	8	18	28	23	18	13	8	4	1	0	4158000	8	3
	(010000112)	9	18	28	23	18	13	8	4	1	-2	3303300	44	8
	(100000022)	9	19	29	24	19	14	9	4	1	-4	182952	192	9
	(101100003)	8	17	26	21	17	13	9	5	1	2	18018000	1	1
	(000200003)	9	18	27	21	17	13	9	5	1	0	2354352	8	1
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	(001010003)	9	18	27	22	17	13	9	5	1	-2	3439800	44	6
	(200001003)	8	18	28	23	18	13	9	5	1	-2	1633500	44	6
	(010001003)	9	18	28	23	18	13	9	5	1	-4	1287000	192	13
	(100000103)	9	19	29	24	19	14	9	5	1	-6	161700	726	20
	(000000013)	10	20	30	25	20	15	10	5	1	-8	5148	2464	13
	(000101110)	9	18	27	21	16	11	7	4	2	2	33033000	1	1
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	(001000210)	9	18	27	22	17	12	7	4	2	0	8494200	8	2
	(101010020)	8	17	26	21	16	12	8	4	2	2	79279200	1	1
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	(110001020)	8	17	27	22	17	12	8	4	2	0	25225200	8	3
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	(200000120)	8	18	28	23	18	13	8	4	2	-2	2102100	44	6
	(010000120)	9	18	28	23	18	13	8	4	2	-4	1667952	192	12
	(100000030)	9	19	29	24	19	14	9	4	2	-6	85800	727	13
	(000021001)	9	18	27	21	15	11	8	5	2	2	10702692	1	1
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(000110101)	9	18	27	21	16	12	8	5	2	-2	14157000	44	8
(300001101)	7	17	27	22	17	12	8	5	2	2	18018000	1	1
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(001001101)	9	18	27	22	17	12	8	5	2	-4	8494200	192	19
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(000200011)	9	18	27	21	17	13	9	5	2	-4	3303300	192	12
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(210011000)	7	16	26	21	16	12	9	6	3	2	24972948	1	1
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(001010100)	9	18	27	22	17	13	9	6	3	-8	1524600	2472	54
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	(010001100)	9	18	28	23	18	13	9	6	3	-10	508200	7749	91
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	(130000010)	7	15	26	22	18	14	10	6	3	2	1849848	1	1
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	(002000001)	9	18	27	23	19	15	11	7	3	-12	46200	22725	76
	(200100001)	8	18	28	23	19	15	11	7	3	-12	85800	22725	134
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	(500000000)	6	17	28	24	20	16	12	8	4	0	2002	8	0
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(300010012)	9	21	33	27	21	16	11	6	2	-4	44594550	192	22
(110010012)	10	21	33	27	21	16	11	6	2	-8	61776000	2472	160
(001010012)	11	22	33	27	21	16	11	6	2	-10	19864845	7747	211
(200001012)	10	22	34	28	22	16	11	6	2	-10	9021375	7747	195
(010001012)	11	22	34	28	22	16	11	6	2	-12	7078500	22712	365
(100000112)	11	23	35	29	23	17	11	6	2	-14	800415	63020	400
(000000022)	12	24	36	30	24	18	12	6	2	-16	19305	166840	160
(130000003)	9	19	32	27	22	17	12	7	2	0	8870400	8	2
(401000003)	8	20	32	27	22	17	12	7	2	2	9626760	1	1
(211000003)	9	20	32	27	22	17	12	7	2	-4	15855840	192	19

(021000003)	10	20	32	27	22	17	12	7	2	-6	8004150	727	30
(102000003)	10	21	32	27	22	17	12	7	2	-8	6486480	2472	60
(300100003)	9	21	33	27	22	17	12	7	2	-6	6195200	727	31
(110100003)	10	21	33	27	22	17	12	7	2	-10	8258250	7747	178
(001100003)	11	22	33	27	22	17	12	7	2	-12	2446080	22712	192
(200010003)	10	22	34	28	22	17	12	7	2	-12	2041875	22712	215
(010010003)	11	22	34	28	22	17	12	7	2	-14	1576575	63020	387
(100001003)	11	23	35	29	23	17	12	7	2	-16	316800	166840	475
(000000103)	12	24	36	30	24	18	12	7	2	-18	17160	424161	289
(100102100)	10	21	32	25	19	13	9	6	3	2	443963520	1	1
(000012100)	11	22	33	26	19	13	9	6	3	0	42046290	8	2
(011010200)	10	20	31	25	19	14	9	6	3	2	927566640	1	1
(100110200)	10	21	32	25	19	14	9	6	3	0	407721600	8	3
(000020200)	11	22	33	26	19	14	9	6	3	-2	31145400	44	5
(210001200)	9	20	32	26	20	14	9	6	3	2	367867500	1	1
(020001200)	10	20	32	26	20	14	9	6	3	0	196756560	8	2
(101001200)	10	21	32	26	20	14	9	6	3	-2	218513295	44	9
(000101200)	11	22	33	26	20	14	9	6	3	-4	38931750	192	13
(300000300)	9	21	33	27	21	15	9	6	3	0	18918900	8	1
(110000300)	10	21	33	27	21	15	9	6	3	-4	26906880	192	13
(001000300)	11	22	33	27	21	15	9	6	3	-6	9084075	727	20
(100030010)	10	21	32	25	18	14	10	6	3	2	163513350	1	1
(010201010)	10	20	31	24	19	14	10	6	3	2	946890945	1	1
(201011010)	9	20	31	25	19	14	10	6	3	2	1630102320	1	1
(011011010)	10	20	31	25	19	14	10	6	3	0	1073572500	8	4
(100111010)	10	21	32	25	19	14	10	6	3	-2	465104640	44	12
(000021010)	11	22	33	26	19	14	10	6	3	-4	34401510	192	12
(210002010)	9	20	32	26	20	14	10	6	3	0	345779280	8	3
(020002010)	10	20	32	26	20	14	10	6	3	-2	184459275	44	8
(101002010)	10	21	32	26	20	14	10	6	3	-4	203483280	192	24
(000102010)	11	22	33	26	20	14	10	6	3	-6	35675640	727	35
(003000110)	10	20	30	25	20	15	10	6	3	2	210420210	1	1
(120100110)	9	19	31	25	20	15	10	6	3	2	1061424000	1	1
(201100110)	9	20	31	25	20	15	10	6	3	0	846806400	8	4
(011100110)	10	20	31	25	20	15	10	6	3	-2	549669120	44	12
(100200110)	10	21	32	25	20	15	10	6	3	-4	190765575	192	23
(210010110)	9	20	32	26	20	15	10	6	3	-2	442047375	44	13
(020010110)	10	20	32	26	20	15	10	6	3	-4	234234000	192	25
(101010110)	10	21	32	26	20	15	10	6	3	-6	252277740	727	81
(000110110)	11	22	33	26	20	15	10	6	3	-8	41291250	2472	94
(300001110)	9	21	33	27	21	15	10	6	3	-4	51744000	192	20
(110001110)	10	21	33	27	21	15	10	6	3	-8	72837765	2472	143
(001001110)	11	22	33	27	21	15	10	6	3	-10	24161280	7749	195
(200000210)	10	22	34	28	22	16	10	6	3	-10	4624620	7747	114
(010000210)	11	22	34	28	22	16	10	6	3	-12	3648645	22725	213
(121000020)	9	19	31	26	21	16	11	6	3	0	141891750	8	3
(202000020)	9	20	31	26	21	16	11	6	3	-2	91306600	44	8
(012000020)	10	20	31	26	21	16	11	6	3	-4	57387330	192	15
(400100020)	8	20	32	26	21	16	11	6	3	2	61256250	1	1
(210100020)	9	20	32	26	21	16	11	6	3	-4	110261250	192	24
(020100020)	10	20	32	26	21	16	11	6	3	-6	57493800	727	44
(101100020)	10	21	32	26	21	16	11	6	3	-8	57807750	2472	113
(000200020)	11	22	33	26	21	16	11	6	3	-10	7432425	7749	96

(300010020)	9	21	33	27	21	16	11	6	3	-6	23821875	727	38
(110010020)	10	21	33	27	21	16	11	6	3	-10	32947200	7749	237
(001010020)	11	22	33	27	21	16	11	6	3	-12	10570560	22725	286
(200001020)	10	22	34	28	22	16	11	6	3	-12	4729725	22725	257
(010001020)	11	22	34	28	22	16	11	6	3	-14	3706560	63085	480
(100000120)	11	23	35	29	23	17	11	6	3	-16	405405	167099	468
(000000030)	12	24	36	30	24	18	12	6	3	-18	9075	425058	168
(002110001)	10	20	30	24	19	15	11	7	3	2	260134875	1	1
(200210001)	9	20	31	24	19	15	11	7	3	2	330330000	1	1
(010210001)	10	20	31	24	19	15	11	7	3	0	228918690	8	3
(120020001)	9	19	31	25	19	15	11	7	3	2	383418750	1	1
(201020001)	9	20	31	25	19	15	11	7	3	0	321080760	8	3
(011020001)	10	20	31	25	19	15	11	7	3	-2	210231450	44	9
(100120001)	10	21	32	25	19	15	11	7	3	-4	87816960	192	19
(000030001)	11	22	33	26	19	15	11	7	3	-6	5945940	726	15
(112001001)	9	19	30	25	20	15	11	7	3	2	566846280	1	1
(003001001)	10	20	30	25	20	15	11	7	3	0	105705600	8	2
(120101001)	9	19	31	25	20	15	11	7	3	0	521756235	8	4
(201101001)	9	20	31	25	20	15	11	7	3	-2	412818120	44	13
(011101001)	10	20	31	25	20	15	11	7	3	-4	267071805	192	31
(100201001)	10	21	32	25	20	15	11	7	3	-6	91069440	727	51
(400011001)	8	20	32	26	20	15	11	7	3	2	107627520	1	1
(210011001)	9	20	32	26	20	15	11	7	3	-4	200154240	192	31
(020011001)	10	20	32	26	20	15	11	7	3	-6	105705600	727	54
(101011001)	10	21	32	26	20	15	11	7	3	-8	112907520	2472	156
(000111001)	11	22	33	26	20	15	11	7	3	-10	18120960	7747	159
(300002001)	9	21	33	27	21	15	11	7	3	-6	18498480	727	32
(110002001)	10	21	33	27	21	15	11	7	3	-10	25872000	7749	200
(001002001)	11	22	33	27	21	15	11	7	3	-12	8494200	22725	247
(311000101)	8	19	31	26	21	16	11	7	3	2	192192000	1	1
(121000101)	9	19	31	26	21	16	11	7	3	-2	162662500	44	12
(202000101)	9	20	31	26	21	16	11	7	3	-4	104247000	192	24
(012000101)	10	20	31	26	21	16	11	7	3	-6	65405340	727	51
(400100101)	8	20	32	26	21	16	11	7	3	0	69369300	8	3
(210100101)	9	20	32	26	21	16	11	7	3	-6	124385625	727	80
(020100101)	10	20	32	26	21	16	11	7	3	-8	64744680	2472	124
(101100101)	10	21	32	26	21	16	11	7	3	-10	64833615	7749	324
(000200101)	11	22	33	26	21	16	11	7	3	-12	8258250	22725	239
(300010101)	9	21	33	27	21	16	11	7	3	-8	26061750	2472	105
(110010101)	10	21	33	27	21	16	11	7	3	-12	35925120	22725	622
(001010101)	11	22	33	27	21	16	11	7	3	-14	11467170	63085	727
(200001101)	10	22	34	28	22	16	11	7	3	-14	4802490	63085	601
(010001101)	11	22	34	28	22	16	11	7	3	-16	3753750	167116	1065
(100000201)	11	23	35	29	23	17	11	7	3	-18	330330	425156	808
(320000011)	8	19	32	27	22	17	12	7	3	0	16558080	8	3
(130000011)	9	19	32	27	22	17	12	7	3	-4	12882870	192	17
(401000011)	8	20	32	27	22	17	12	7	3	-2	13939200	44	8
(211000011)	9	20	32	27	22	17	12	7	3	-8	22880000	2472	132
(021000011)	10	20	32	27	22	17	12	7	3	-10	11531520	7747	180
(102000011)	10	21	32	27	22	17	12	7	3	-12	9313920	22725	351
(300100011)	9	21	33	27	22	17	12	7	3	-10	8848125	7749	189
(110100011)	10	21	33	27	22	17	12	7	3	-14	11760000	63085	977
(001100011)	11	22	33	27	22	17	12	7	3	-16	3468465	167116	986

(200010011)	10	22	34	28	22	17	12	7	3	-16	2851200	167116	1091
(010010011)	11	22	34	28	22	17	12	7	3	-18	2196480	425156	1854
(100001011)	11	23	35	29	23	17	12	7	3	-20	424710	1043926	2076
(000000111)	12	24	36	30	24	18	12	7	3	-22	21120	2483871	1071
(410000002)	8	20	33	28	23	18	13	8	3	-4	755040	192	10
(220000002)	9	20	33	28	23	18	13	8	3	-10	962280	7747	96
(030000002)	10	20	33	28	23	18	13	8	3	-12	450450	22712	104
(301000002)	9	21	33	28	23	18	13	8	3	-12	847000	22725	185
(111000002)	10	21	33	28	23	18	13	8	3	-16	1034880	167099	797
(002000002)	11	22	33	28	23	18	13	8	3	-18	240240	425156	591
(200100002)	10	22	34	28	23	18	13	8	3	-18	444675	425156	1027
(010100002)	11	22	34	28	23	18	13	8	3	-20	331695	1043926	1608
(100010002)	11	23	35	29	23	18	13	8	3	-22	107250	2483970	2033
(000001002)	12	24	36	30	24	18	13	8	3	-24	9240	5746226	1230
(111200000)	9	19	30	24	20	16	12	8	4	2	63423360	1	1
(002200000)	10	20	30	24	20	16	12	8	4	0	12458160	8	1
(200300000)	9	20	31	24	20	16	12	8	4	0	14864850	8	1
(010300000)	10	20	31	24	20	16	12	8	4	-2	10193040	44	4
(112010000)	9	19	30	25	20	16	12	8	4	0	63423360	8	3
(003010000)	10	20	30	25	20	16	12	8	4	-2	11679525	44	4
(310110000)	8	19	31	25	20	16	12	8	4	2	62432370	1	1
(120110000)	9	19	31	25	20	16	12	8	4	-2	54362880	44	8
(201110000)	9	20	31	25	20	16	12	8	4	-4	42282240	192	19
(011110000)	10	20	31	25	20	16	12	8	4	-6	27181440	727	38
(100210000)	10	21	32	25	20	16	12	8	4	-8	8918910	2472	50
(400020000)	8	20	32	26	20	16	12	8	4	0	8828820	8	1
(210020000)	9	20	32	26	20	16	12	8	4	-6	16186170	727	31
(020020000)	10	20	32	26	20	16	12	8	4	-8	8494200	2472	42
(101020000)	10	21	32	26	20	16	12	8	4	-10	8918910	7747	116
(000120000)	11	22	33	26	20	16	12	8	4	-12	1372140	22712	94
(040001000)	9	18	31	26	21	16	12	8	4	2	10020010	1	1
(311001000)	8	19	31	26	21	16	12	8	4	0	37065600	8	3
(121001000)	9	19	31	26	21	16	12	8	4	-4	31216185	192	19
(202001000)	9	20	31	26	21	16	12	8	4	-6	19819800	727	36
(012001000)	10	20	31	26	21	16	12	8	4	-8	12387375	2472	61
(400101000)	8	20	32	26	21	16	12	8	4	-2	12899250	44	6
(210101000)	9	20	32	26	21	16	12	8	4	-8	22934340	2472	99
(020101000)	10	20	32	26	21	16	12	8	4	-10	11891880	7747	143
(101101000)	10	21	32	26	21	16	12	8	4	-12	11797500	22725	337
(000201000)	11	22	33	26	21	16	12	8	4	-14	1470150	63085	225
(300011000)	9	21	33	27	21	16	12	8	4	-10	4414410	7749	108
(110011000)	10	21	33	27	21	16	12	8	4	-14	6040320	63085	558
(001011000)	11	22	33	27	21	16	12	8	4	-16	1905750	167116	594
(200002000)	10	22	34	28	22	16	12	8	4	-16	630630	167133	379
(010002000)	11	22	34	28	22	16	12	8	4	-18	490050	425227	652
(320000100)	8	19	32	27	22	17	12	8	4	-2	5780775	44	6
(130000100)	9	19	32	27	22	17	12	8	4	-6	4484480	726	25
(401000100)	8	20	32	27	22	17	12	8	4	-4	4826250	192	13
(211000100)	9	20	32	27	22	17	12	8	4	-10	7882875	7747	177
(021000100)	10	20	32	27	22	17	12	8	4	-12	3963960	22712	223
(102000100)	10	21	32	27	22	17	12	8	4	-14	3185325	63085	421
(300100100)	9	21	33	27	22	17	12	8	4	-12	2995200	22725	232
(110100100)	10	21	33	27	22	17	12	8	4	-16	3963960	167116	1114

	(001100100)	11	22	33	27	22	17	12	8	4	-18	1161600	425227	1064
	(200010100)	10	22	34	28	22	17	12	8	4	-18	926640	425227	1141
	(010010100)	11	22	34	28	22	17	12	8	4	-20	711480	1044218	1865
	(100001100)	11	23	35	29	23	17	12	8	4	-22	126720	2485020	1835
	(000000200)	12	24	36	30	24	18	12	8	4	-24	4950	5749565	694
	(410000010)	8	20	33	28	23	18	13	8	4	-6	606375	727	22
	(220000010)	9	20	33	28	23	18	13	8	4	-12	770770	22712	171
	(030000010)	10	20	33	28	23	18	13	8	4	-14	360360	63020	194
	(301000010)	9	21	33	28	23	18	13	8	4	-14	675675	63085	342
	(111000010)	10	21	33	28	23	18	13	8	4	-18	823680	425156	1382
	(002000010)	11	22	33	28	23	18	13	8	4	-20	190575	1044218	973
	(200100010)	10	22	34	28	23	18	13	8	4	-20	351000	1044218	1717
	(010100010)	11	22	34	28	23	18	13	8	4	-22	261360	2485020	2674
	(100010010)	11	23	35	29	23	18	13	8	4	-24	83160	5749818	3208
	(000001010)	12	24	36	30	24	18	13	8	4	-26	6930	12970045	1843
	(500000001)	8	21	34	29	24	19	14	9	4	-8	19305	2472	15
	(310000001)	9	21	34	29	24	19	14	9	4	-16	49280	167116	295
	(120000001)	10	21	34	29	24	19	14	9	4	-20	47190	1043926	857
	(201000001)	10	22	34	29	24	19	14	9	4	-22	45045	2485020	1635
	(011000001)	11	22	34	29	24	19	14	9	4	-24	31185	5749818	2255
	(100100001)	11	23	35	29	24	19	14	9	4	-26	17280	12971009	3295
	(000010001)	12	24	36	30	24	19	14	9	4	-28	2310	28592513	2095
	(400000000)	9	22	35	30	25	20	15	10	5	-18	715	425227	90
	(210000000)	10	22	35	30	25	20	15	10	5	-24	1485	5750072	688
	(020000000)	11	22	35	30	25	20	15	10	5	-26	825	12971009	744
	(101000000)	11	23	35	30	25	20	15	10	5	-28	990	28595548	1628
	(000100000)	12	24	36	30	25	20	15	10	5	-30	210	61721165	1168

Table 2: A_{10} representations in E_{11} up to level $\ell = 10$

ℓ	p	m										Λ^2	dim $\mathcal{R}(\Lambda)$	mult(Λ)	μ
1	(0010000000)	0	0	0	0	0	0	0	0	0	0	2	165	1	1
2	(0000010000)	1	2	3	2	1	0	0	0	0	0	2	462	1	1
3	(1000000100)	1	3	5	4	3	2	1	0	0	0	2	1760	1	1
	(0000000010)	2	4	6	5	4	3	2	1	0	0	0	55	8	0
4	(0010000010)	2	4	6	5	4	3	2	1	0	0	2	8470	1	1
	(2000000001)	1	4	7	6	5	4	3	2	1	0	2	715	1	1
	(0100000001)	2	4	7	6	5	4	3	2	1	0	0	594	8	0
	(1000000000)	2	5	8	7	6	5	4	3	2	1	-2	11	46	1
5	(0000010010)	3	6	9	7	5	3	2	1	0	0	2	20328	1	1
	(1001000001)	2	5	8	6	5	4	3	2	1	0	2	33033	1	1
	(0000100001)	3	6	9	7	5	4	3	2	1	0	0	4752	8	0
	(0200000000)	2	4	8	7	6	5	4	3	2	1	2	1210	0	0
	(1010000000)	2	5	8	7	6	5	4	3	2	1	0	1485	8	1
	(0001000000)	3	6	9	7	6	5	4	3	2	1	-2	330	46	1
6	(1000000110)	3	7	11	9	7	5	3	1	0	0	2	57200	1	1
	(0000000020)	4	8	12	10	8	6	4	2	0	0	0	1210	8	0
	(0100010001)	3	6	10	8	6	4	3	2	1	0	2	214500	1	1
	(1000001001)	3	7	11	9	7	5	3	2	1	0	0	33033	8	1
	(0000000101)	4	8	12	10	8	6	4	2	1	0	-2	1485	44	1
	(0011000000)	3	6	9	7	6	5	4	3	2	1	2	29040	1	1
	(2000100000)	2	6	10	8	6	5	4	3	2	1	2	25740	1	1
	(0100100000)	3	6	10	8	6	5	4	3	2	1	0	20328	8	0
	(1000010000)	3	7	11	9	7	5	4	3	2	1	-2	4752	46	2
	(0000001000)	4	8	12	10	8	6	4	3	2	1	-4	330	206	1
7	(0010000020)	4	8	12	10	8	6	4	2	0	0	2	176176	1	1
	(0001001001)	4	8	12	9	7	5	3	2	1	0	2	755040	1	1
	(1100000101)	3	7	12	10	8	6	4	2	1	0	2	582120	1	1
	(0010000101)	4	8	12	10	8	6	4	2	1	0	0	212355	8	1
	(2000000011)	3	8	13	11	9	7	5	3	1	0	0	27720	8	1
	(0100000011)	4	8	13	11	9	7	5	3	1	0	-2	22880	44	2
	(1000000002)	4	9	14	12	10	8	6	4	2	0	-4	715	192	1
	(0000200000)	4	8	12	9	6	5	4	3	2	1	2	60984	0	0
	(1010010000)	3	7	11	9	7	5	4	3	2	1	2	495495	1	1
	(0001010000)	4	8	12	9	7	5	4	3	2	1	0	98010	8	1
	(1100001000)	3	7	12	10	8	6	4	3	2	1	0	125840	8	1
	(0010001000)	4	8	12	10	8	6	4	3	2	1	-2	45375	46	2
	(2000000100)	3	8	13	11	9	7	5	3	2	1	-2	10296	46	3
	(0100000100)	4	8	13	11	9	7	5	3	2	1	-4	8470	206	3
	(1000000010)	4	9	14	12	10	8	6	4	2	1	-6	594	801	4
	(0000000001)	5	10	15	13	11	9	7	5	3	1	-8	11	2801	1
8	(0000010020)	5	10	15	12	9	6	4	2	0	0	2	377520	1	1
	(0000002001)	5	10	15	12	9	6	3	2	1	0	2	283140	1	1
	(1000100101)	4	9	14	11	8	6	4	2	1	0	2	4802490	1	1
	(0000010101)	5	10	15	12	9	6	4	2	1	0	0	424710	8	1
	(0110000011)	4	8	13	11	9	7	5	3	1	0	2	2081079	1	1
	(1001000011)	4	9	14	11	9	7	5	3	1	0	0	1182720	8	2
	(0000100011)	5	10	15	12	9	7	5	3	1	0	-2	165165	44	2
	(2100000002)	3	8	14	12	10	8	6	4	2	0	2	133650	1	1
	(0200000002)	4	8	14	12	10	8	6	4	2	0	0	75075	8	0
	(1010000002)	4	9	14	12	10	8	6	4	2	0	-2	91476	44	2

	(0001000002)	5	10	15	12	10	8	6	4	2	0	-4	20020	192	2
	(1000020000)	4	9	14	11	8	5	4	3	2	1	2	594594	1	1
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	(2010000100)	3	8	13	11	9	7	5	3	2	1	2	1081080	1	1
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	(1001000100)	4	9	14	11	9	7	5	3	2	1	-2	424710	46	4
	(0000100100)	5	10	15	12	9	7	5	3	2	1	-4	58080	206	3
	(2100000010)	3	8	14	12	10	8	6	4	2	1	0	110110	8	1
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	(3000000001)	3	9	15	13	11	9	7	5	3	1	-2	3080	46	2
	(1100000001)	4	9	15	13	11	9	7	5	3	1	-6	4719	801	7
	(0010000001)	5	10	15	13	11	9	7	5	3	1	-8	1760	2801	6
	(2000000000)	4	10	16	14	12	10	8	6	4	2	-8	66	2821	3
	(0100000000)	5	10	16	14	12	10	8	6	4	2	-10	55	9071	5
9	(1000000120)	5	11	17	14	11	8	5	2	0	0	2	880880	1	1
	(0000000030)	6	12	18	15	12	9	6	3	0	0	0	15730	8	0
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	(2000000110)	5	12	19	16	13	10	7	4	2	1	-6	330330	801	14
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	(1000000020)	6	13	20	17	14	11	8	5	2	1	-10	12870	8885	17
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	(1000001000)	6	13	20	17	14	11	8	6	4	2	-14	3465	77667	38
	(0000000100)	7	14	21	18	15	12	9	6	4	2	-16	165	210723	16

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